Real-time Buried Object Detection Using LMMSE Estimation

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Abstract—We present the application of linear minimum mean square error (LMMSE) estimation to GPR data for achieving buried object detection. Without employing any empirical assumptions, nonstationary form of Wiener-Hopf equations is applied to GPR signals to estimate the next sample in normal conditions. A large deviation from this estimation indicates the presence of a buried object. The technique is causal, which allows it to be used in real-time applications. Our approach is theoretically optimal in linear minimum mean square error sense, and it is also validated with the tests that are carried out on a comprehensive data set of GPR signals.

I. INTRODUCTION

Nonintrusive detection of buried objects appeals great interest among scientists. Among many available methods, ground penetrating radar (GPR) has become especially popular. GPR has a wide spectrum of application areas, some of which include archaeological investigations, building condition assessment, forensic investigations, detection of buried mines, road condition survey and pipe detection [1].

There are two basic factors, by means of which GPR has gained its reputation. First, GPR is capable of sensing both metallic and nonmetallic objects as it is sensitive to all three characteristics of the scanned area, which are, electric permittivity, electric conductivity and magnetic permeability. Moreover, GPR can survey an area before the sensor moves past over it. This is extremely beneficial in detection of dangerous objects, such as buried landmines [2]. A simplistic GPR block diagram is provided in Fig. 1, where antenna coupling, ground bounce and target signature are shown.

In this study, we propose to use time domain linear minimum mean square error (LMMSE) estimation in the buried object detection problem. Actually, this implementation turns out to be a Wiener filter, without the stationarity assumption of the signals. LMMSE estimation is vastly exploited in the communications society, however, to the best of our knowledge, it is not used in the buried object detection problem using GPR yet. In [3], baseband Wiener processing is applied to laser induced acoustic scanner data, by assuming a multipath model for the received signal. In [4], Wiener filter is used to detect changes in synthetic aperture radar images. Neither of these studies deals with GPR data. Our technique depends on estimating the next sample in the GPR signal using the optimal LMMSE estimator. This estimator corresponds to a causal FIR filter, which allows the method to be applied in real-time applications. After estimating the next sample, the sample is gathered and compared with its estimation. A large deviation from the estimated value signifies an anomaly at the inspected zone.

This paper is structured as follows. Section II discusses the problem formulation and presents the proposed method. Section III gives the results of applying the technique to an extensive GPR data set, which is collected using a GPR sensor developed at The Scientific and Technological Research Council of Turkey (TÜBİTAK). Conclusion is drawn in Section IV, which finalizes our discussion.

II. PROBLEM FORMULATION AND ALGORITHM

A. Data Representation

A GPR B-scan is represented as an ensemble of A-scans. Let a B-scan be composed of $r$ A-scans, each of which contains $q$ samples. We model this B-scan as $r$ realizations of $q$ random sequences. In other words, each depth bin forms a random sequence.

For representing each random sequence, we utilize the fact that, clutter signatures are similar among themselves, however, they exhibit a high degree of discrepancy from buried object signatures. In other words, in the absence of a buried object, GPR signals vary slowly. Hence, clutter samples can be represented as a linear combination of previous samples. This assumption is used in quite a few of previous studies (e.g. [5], [6]). Under this assumption, the linear combination model can be shown as

![Fig. 1. Main structure of the GPR system](image-url)
\[
\hat{X}(n) = \sum_{k=1}^{N_{\text{tap}}} w(k) X(n - k)
\]  
(1)

where \(X\) is a random sequence, \(\hat{X}\) is the estimator of this sequence and \(w(k)\) values are the coefficients of each sample. As it can be seen, estimated value depends on \(N_{\text{tap}}\) previous values of the random sequence, which makes the system causal.

**B. Preprocessing**

For clutter reduction, an averaged value of an ensemble of A-scans is subtracted from each A-scan [1]. Evidently, this ensemble should not include contributions from target \(s\). For this purpose, first \(N_{\text{tap}}\) A-scans are used in the clutter reduction step at each scan. Clearly, \(N_{\text{tap}}\) is much smaller than the total number of A-scans.

**C. Detection Algorithm**

We seek the \(w(k)\) coefficients in (1) such that 
\[
E \{ (X(n) - \hat{X}(n))^2 \} \text{ is minimized.}
\]

This is the expected value of the mean square error, and it can be shown as

\[
MSE = E \left\{ \left( X(n) - \sum_{k=1}^{N_{\text{tap}}} w(k)X(n - k) \right)^2 \right\}.
\]  
(2)

To minimize \(MSE\), this expression is differentiated with respect to each \(w(i), i = 1, 2, ..., N_{\text{tap}}\). The \(w(i)\) values correspond to the coefficients of an FIR filter with \(N_{\text{tap}}\) taps.

\[
E \left\{ -2 \left( X(n) - \sum_{k=1}^{N_{\text{tap}}} w(k)X(n - k) \right)X(n - i) \right\} = 0
\]  
(3)

Note that (3) states that the error is orthogonal to each observation [7]. Using the linearity of the expected value operator, we get

\[
E \{ X(n)X(n - i) \} = \sum_{k=1}^{N_{\text{tap}}} E \{ w(k)X(n - i)X(n - k) \}.
\]  
(4)

Writing the expected values in the form of autocorrelations results in

\[
R_{XX}(n, n - i) = \sum_{k=1}^{N_{\text{tap}}} w(k)R_{XX}(n - i, n - k).
\]  
(5)

This set of equations is called the nonstationary Wiener-Hopf equations. We can also write this set of equations in matrix form as

\[
w = R_{XX}^{-1}P
\]  
(6)

where \(R_{XX}\) is the autocorrelation matrix of past samples and \(P\) is the crosscorrelation vector between the current sample and the past samples. The solution of this matrix equation gives the optimal filter coefficients. After calculating the optimal filter coefficients, the LMMSE estimate of the current sample is found using (1). The detection function is the norm of the difference between the estimated A-scan and the observed A-scan. A block diagram of this system is given in Fig. 2.

![Block diagram of the proposed method](image)

Note that this block diagram exhibits the process for a single sample of an A-scan. This operation is carried out at each depth bin and resulting error values are summed up to form the total estimation error.

As shown in (6), to find the optimal filter coefficients, autocorrelation matrix of the past samples must be inverted. It is found out that this matrix is rank deficient in most practical cases. So, the Moore-Penrose pseudoinverse is used in the calculations. The Moore-Penrose pseudoinverse finds a least squares solution to the matrix inversion problem. The Moore-Penrose pseudoinverse of an \(m \times n\) matrix \(G\) is a unique \(n \times m\) matrix \(G^+\) satisfying the four Penrose equations [8], which are given as

\[
GG^+G = G
\]
\[
G^+GG = G^+
\]
\[
(GG^+)^H = GG^+
\]
\[
(G^+G)^H = G^+G.
\]  
(7)

In this set of equations, superscript \(H\) denotes the Hermitian operator. It should be emphasized that the Moore-Penrose pseudoinverse agrees with the inverse \(G^{-1}\) of \(G\) when \(G\) has full rank [9]. Hence, using the Moore-Penrose pseudoinverse stabilizes the calculation of (6) in case of singular \(G\), and does not introduce any error when \(G\) is nonsingular.

**III. Experimental Results**

This section presents the detection results for the proposed LMMSE estimation approach. The tests are performed under 3 different soil types. In all cases, soil is dry and relative static permittivity values of the soil vary between 2 and 3.
The total number of target signatures (B-scans) is 518. Each target signature consists of 240 A-scans having 256 samples. The buried objects are very diverse, including small plastic anti-personnel mine equivalents, large metal anti-tank mine equivalents, glass bottles, tin cans, stones and oil bins. The number of each of these objects is given in Table I.

<table>
<thead>
<tr>
<th>Object type</th>
<th>Number of objects</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surrogate M-15 mine</td>
<td>22</td>
</tr>
<tr>
<td>Surrogate M6A2 mine</td>
<td>95</td>
</tr>
<tr>
<td>Surrogate M7A2 mine</td>
<td>44</td>
</tr>
<tr>
<td>Surrogate PMD mine</td>
<td>22</td>
</tr>
<tr>
<td>Surrogate PMN mine</td>
<td>22</td>
</tr>
<tr>
<td>Surrogate TM62M mine</td>
<td>22</td>
</tr>
<tr>
<td>Surrogate TS-50 mine</td>
<td>114</td>
</tr>
<tr>
<td>Surrogate VS-50 mine</td>
<td>24</td>
</tr>
<tr>
<td>Glass bottle</td>
<td>68</td>
</tr>
<tr>
<td>Oil bin</td>
<td>42</td>
</tr>
<tr>
<td>Stone</td>
<td>22</td>
</tr>
<tr>
<td>Tin can</td>
<td>21</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td><strong>518</strong></td>
</tr>
</tbody>
</table>

We present 3 detection examples in this section. The examples include a surrogate M-15 mine, a surrogate TS-50 mine and a glass bottle.

In the first case, a surrogate M-15 mine is under inspection. The buried object has very similar properties with an M-15 mine, in terms of both shape and content. M-15 is a large circular anti-tank mine, which has a steel case. It has a height of 150 mm and a diameter of 333 mm. Burial depth of the object is 300 mm. In this example, $N_{tap}$ is taken to be 10, and the detection threshold is 0.6. Detection alarm is given in the shaded area. The B-scan and corresponding detection function are given in Fig. 3. For a more rigorous inspection of the effect of threshold on detection, a receiver-operating characteristics curve is given at the end of this section.

Second example involves a surrogate TS-50 mine. TS-50 is a small circular minimum metal anti-personnel mine, and contains almost no metal at all. Its height is 45 mm and its diameter is 90 mm. The buried mine equivalent has the exact same characteristics. The burial depth is 70 mm in this case. In this case, $N_{tap}$ and the detection threshold are the same as the previous case. The result is given in Fig. 4. Again, the shaded area shows the detection region, which is correct.

In the last example, a glass bottle is buried to a depth of 90 mm. The bottle has a height of 180 mm and the diameter of its largest cross section is 40 mm. In this last example, $N_{tap}$ and the detection threshold are the same as the previous cases. The detection result is given in Fig. 5. This result clearly shows that GPR is capable of sensing objects with no metal content.
Receiver operating characteristic curves are given in the next figure. As explained before, 12 different types of objects were buried under 3 different soil conditions. Total number of target signatures is 518. For generating the curves, the detection threshold is increased from 0.5 to 1 with 0.01 resolution. The proposed approach is tested with 3 different tap numbers.

As it is evident from Fig. 6, increasing $N_{\text{tap}}$ from 3 to 10 provides a significant improvement on the performance of the algorithm. However, an increase from 10 to 15 is not that effective. Although there is a slight amendment, ROC curves for $N_{\text{tap}} = 10$ and $N_{\text{tap}} = 15$ coincide for most of the threshold values. This shows that the FIR filter has a convergence point, and further increase in the number of taps is not preferable as the computational complexity of the system increases. This increase in complexity is very momentous for our approach, as the algorithm requires a matrix inversion for each sample of each A-scan. And the matrix to be inverted is of size $N_{\text{tap}}$ by $N_{\text{tap}}$. Hence, one should lower $N_{\text{tap}}$ as much as possible, while abstaining from considerable performance degradation.

IV. CONCLUSION

This paper proposed the application of linear minimum mean square error estimation to the buried object detection problem using GPR. As the GPR signals are slowly varying in the absence of a buried object, the signals at each depth bin are modeled as a linear combination of the previous samples. The aim of the detection algorithm is basically to find the coefficients of the previous samples in this signal model. The detection function is computed as the norm of the difference of the estimated signal and the measured signal. The normalized test statistic is compared to fixed threshold values, and ROC curves for different window sizes are given. An overall detection rate of 94% is achieved with 20% false alarm rate, when the tap number of the filter is selected satisfactorily high.

REFERENCES